SOLUTION OF THE SELF-SIMULATING PROBLEM OF HEAT AND MOISTURE TRANSFER DURING FREEZING OF DISPERSE SOILS

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The article presents the construction of the self-simulating solution of the problem of freezing of disperse soils attended by migration of moisture in the melted and the frozen zones. The conditions are determined under which a freezing layer forms between them. A comparison of the theoretical data with the experimental ones shows qualitative coincidence.

1. Introduction. It is known that the principal processes attending the freezing of moist soils are the phase transition of water and mass transfer, and that migration of water is found both in the melted and in the freezing zone. In addition to that, in many experiments it was ascertained that the front of macroscopic ice evolution lags noticeably behind the boundary of incipient freezing. The region between these boundaries is called freezing region in Soviet literature [1, 2], in foreign literature it is called the frozen fringe [3]. The formation of such a zone was observed in soils containing argillaceous particles (loams, clays), and the quantitative characteristics such as the size of this zone, the temperature and moisture content at the boundaries of the onset and completion of the phase transition of the moisture depend on internal parameters of the soil as well as on the external conditions of freezing. In particular, when the initial moisture content of the soil is high, this layer is noticeably smaller, the moisture at the boundary of freezing increases with increasing cooling rate, etc. Ice does not form on the boundary of ice evolution alone but also inside the freezing zone in which the total moisture content noticeably decreases, attaining its minimum on the boundary of ice evolution.

The mathematical model of the process of freezing of disperse soils has to describe all these phenomena. The authors of [4-6] investigated them on the basis of a single mathematical model describing conductive heat transfer: moisture transfer in the melted and freezing zones; phase transition in a broad temperature range; relaxation effects of the processes of crystallization of moisture and of ice melting. An analysis of the solutions obtained with the aid of the finite difference method showed that the suggested method of calculation yields results close to the experimental ones.

The present article describes the self-simulating solution that takes the first three of the above-mentioned processes into account. It should be noted that the existing self-simulating solutions (see, e.g., [7, 8]) were derived on the assumption that there is no moisture transfer in the freezing zone, and that the moisture content on the boundary between the melted and the frozen zones is a constant that was previously specified. The discrepancy between the solutions thus obtained and the experimental data described in a number of publications (see, e.g., [2, 9]) are apparently due to the artificial separation of the melted from the frozen zones. Below, it will be shown that when the existence of an intermediate zone is taken into account, good agreement with the experiments is attained. The suggested model assumes that in the melted and frozen zones the same mechanism of moisture transfer acts. The phenomenological model is general, and in the unidimensional case it has the form $q = -k\, \partial w/\partial x$ [6]. It follows from this relation that the migration of moisture is directed to the side of lower temperatures (see Fig. 1 where curve 1 represents the dependence of the content of nonfrozen water on the temperature for finely disperse soils). In constructing the self-simulating solution the piecewise linear approximation of this dependence (curve 2) is used.

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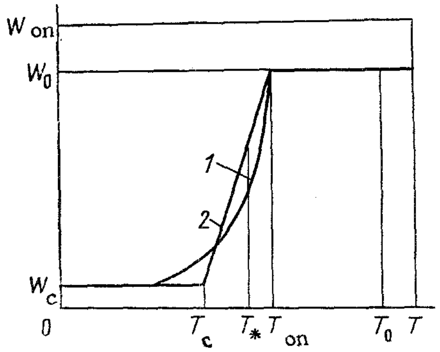

Fig. 1

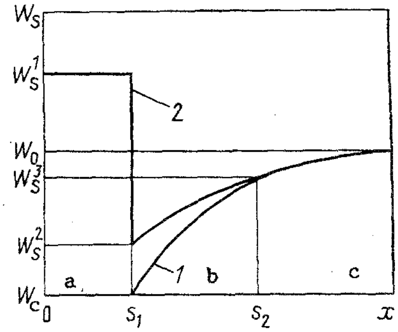

Fig. 2

Fig. 1. Dependence of the equilibrium content of nonfrozen water on the temperature: 1) characteristic curve of finely dispersed soils; 2) approximation of curve 1. The reference point on the axis of abscissas is not 0 but T_x.

Fig. 2. Distribution of moisture content W (1) and of summary moisture W_s (2) along the freezing soil sample.

It is known that the heat and mass transfer coefficients of soils depend largely on parameters such as temperature, moisture content, iciness. For instance, the diffusion coefficient of moisture k increases with increasing moisture content W, and for the different W the values of k may differ from each other by 1-2 orders of magnitude. In the suggested model this is taken into account by assigning in each of the three zones individual values of λ, c, k, differing from each other. In addition, we use some relations satisfied by the heat and mass transfer coefficients of real soils [7]. Denoting with the subscripts 1, 2, 3 the frozen, the freezing, and the melted zone, respectively, we can write the following inequalities:

$$\lambda_3 < \lambda_2,\ k_2 < k_3,\ a_2 < a_3,\ k_3 < a_3. \tag{1}$$

2. Mathematical Statement of the Problem. In self-simulating statement the unidimensional problem of the freezing of disperse soils of Stefan type is formulated as follows. Initially the medium has uniform temperature T_0 and moisture content W_0. At the instant t = 0 the temperature $T_x < 0°C$ is established. The entire investigated region x > 0 is divided into three zones: the frozen zone (a), the freezing zone (b), and the melted zone (c) (Fig. 2).

In the first zone $0 < x < s_1(t)$

$$\frac{\partial T_1}{\partial t} = a_1 \frac{\partial T_1}{\partial x^2},\quad W_1 = W_c; \tag{2}$$

in the second zone $s_1(t) < x < s_2(t)$

$$\frac{\partial T_2}{\partial t} = a_2 \frac{\partial^2 T_2}{\partial x^2},\quad W_2 = W_{on}(T_2) = (T_2 - T_c)\frac{\Delta W}{\Delta T} + W_c; \tag{3}$$

in the third zone $s_2(t) < x < \infty$

$$\frac{\partial T_3}{\partial t} = a_3 \frac{\partial^2 T_3}{\partial x^2},\quad \frac{\partial W_3}{\partial t} = k_3 \frac{\partial^2 W_3}{\partial x^2}. \tag{4}$$

Here,

$$a_1 = \lambda_1/c_1,\quad a_2 = \frac{\lambda_2 + k_2 \varkappa \rho \Delta W/\Delta T}{c_2 + \varkappa \rho \Delta W/\Delta T},\quad a_3 = \lambda_3/c_3. \tag{5}$$

The coefficient a_2 in (3), (5) is determined in the following way. Conductive heat transfer, migration of moisture, and phase transition of water in the freezing zone are described by the relations

$$c_2 \frac{\partial T_2}{\partial t} = \lambda_2 \frac{\partial^2 T_2}{\partial x^2} + \varkappa \rho \frac{\partial L_2}{\partial t},\quad \frac{\partial W_2}{\partial t} + \frac{\partial L_2}{\partial t} = k_2 \frac{\partial^2 W_2}{\partial x^2}.$$

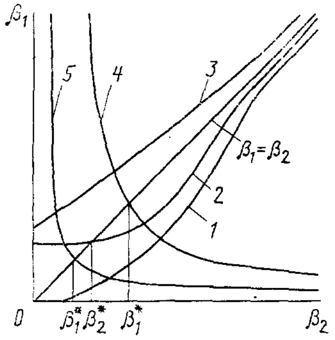

Fig. 3. Correlation between the coefficients β_1, β_2 satisfying: 1) Eq. (17) when condition (20) is fulfilled; 2) (17) with (21) fulfilled; 3) (17) with (22) fulfilled; 4) (16) when $\beta_1^* > \beta_2^*$; 5) (16) when $\beta_1^* < \beta_2^*$.

Since $\partial W_2/\partial t = \Delta W/\Delta T\ \partial T_2/\partial t$, the elimination of $\partial L_2/\partial t$ yields

$$\left(c_2 + \varkappa\rho\frac{\Delta W}{\Delta T}\right)\frac{\partial T_2}{\partial t} = \left(\lambda_2 + k_2\varkappa\rho\frac{\Delta W}{\Delta T}\right)\frac{\partial^2 T_2}{\partial x^2}.$$

At the movable boundaries s_1, s_2 we specify the conditions of continuity of the function of temperature and moisture content, and also of conserving the balances of heat and moisture content.

On

$$s_1 - \quad T_1 = T_2 = T_c \quad W_2 = W_c, \tag{6}$$

$$[\lambda\partial T/\partial x]_1 + \varkappa\rho\,[k\partial W/\partial x]_1 = 0, \tag{7}$$

$$v_1\,[W_s]_1 + [k\partial W/\partial x]_1 = 0, \quad v_1 = ds_1/dt. \tag{8}$$

On

$$s_2 - \quad T_2 = T_3 = T_* \quad W_2 = W_3 = W_{\text{on}}(T_*), \tag{9}$$

$$[\lambda\partial T/\partial x]_2 + \varkappa\rho\,[k\partial W/\partial x]_2 = 0, \tag{10}$$

$$v_2\,[W_s]_2 + [k\partial W/\partial x]_2 = 0, \; v_2 = ds_2/dt.$$

In addition to that, on the second boundary we specify that the moisture flows are continuous, and then condition (10) reduces to

$$[\lambda\partial T/\partial x]_2 = 0, \; [k\partial W/\partial x]_2 = 0. \tag{11}$$

If we introduce the self-simulating variable $\eta = x/2\sqrt{t}$ and represent s_1, s_2 as $s_1 = 2\beta_1\sqrt{t}$, $s_2 = 2\beta_2\sqrt{t}$, then with a view to (2)-(6), (9), the solution of the initial problem is written in the form:

$$\begin{gathered}
T_1 = \frac{T_c - T_x}{\operatorname{erf}(\beta_1/\sqrt{a_1})}\operatorname{erf}(\eta/\sqrt{a_1}) + T_x, \; W_1 = W_c; \\
T_2 = \frac{T_* - T_c}{[\operatorname{erf}(\beta_2/\sqrt{a_2}) - \operatorname{erf}(\beta_1/\sqrt{a_2})]}\,[\operatorname{erf}(\eta/\sqrt{a_2}) - \operatorname{erf}(\beta_1/\sqrt{a_2})] + T_c; \\
W_2 = \Delta W/\Delta T\,(T_2 - T_c) + W_c; \\
T_3 = \frac{T_0 - T_*}{\operatorname{erfc}(\beta_2/\sqrt{a_3})}\,[\operatorname{erf}(\eta/\sqrt{a_3}) - \operatorname{erf}(\beta_2/\sqrt{a_3})] + T_*; \\
W_3 = \frac{W_0 - W_{\text{or}}(T_*)}{\operatorname{erfc}(\beta_2/\sqrt{k_3})}\,[\operatorname{erf}(\eta/\sqrt{k_3}) - \operatorname{erf}(\beta_2/\sqrt{k_3})] + W_{\text{or}}(T_*).
\end{gathered} \tag{12}$$

The distribution profiles of the moisture content and of the summary moisture are presented in Fig. 2. Conditions (7), (11) are written in the form of ratios:

$$\frac{\lambda_1(T_c - T_x)}{\sqrt{a_1}\operatorname{erf}(\beta_1/\sqrt{a_1})}\exp(-\beta_1^2/a_1) = \varkappa_1\frac{\lambda_2(T_* - T_c)}{\sqrt{a_2}\,[\operatorname{erf}(\beta_2/\sqrt{a_2}) - \operatorname{erf}(\beta_1/\sqrt{a_2})]}\exp(-\beta_1^2/a_2), \tag{13}$$

$$\frac{\lambda_2(T_* - T_c)}{\sqrt{a_2}\,[\operatorname{erf}(\beta_2/\sqrt{a_2}) - \operatorname{erf}(\beta_1/\sqrt{a_2})]}\exp(-\beta_2^2/a_2) = \frac{\lambda_3(T_0 - T_*)}{\sqrt{a_3}\operatorname{erfc}(\beta_2/\sqrt{a_3})}\exp(-\beta_2^2/a_3), \tag{14}$$

$$k_2 \frac{T_* - T_c}{\sqrt{a_2}\,[\operatorname{erf}(\beta_2/\sqrt{a_2}) - \operatorname{erf}(\beta_1/\sqrt{a_2})]} \exp(-\beta_2^2/a_2) = \sqrt{k_3} \frac{T_{on} - T_*}{\operatorname{erfc}(\beta_2/\sqrt{k_3})} \exp(-\beta_2^2/k_3). \tag{15}$$

Here,

$$\varkappa_1 = 1 + \frac{k_2 \varkappa \rho}{\lambda_2} \frac{\Delta W}{\Delta T}.$$

3. Criterion of Formation of the Freezing Zone. In deriving the last relations it was understood that the initial parameters are such that the entire domain of solutions is divided into three parts. In fact, it is possible that a situation arises where an intermediate layer does not form, and the frozen zone is in direct contact with the melted one. Below we will obtain a condition such that when it is fulfilled, three zones form, and when it is not fulfilled, there are only two zones. For that we have to regroup the separate expressions in (13)-(15):

$$\frac{\lambda_1 (T_c - T_x)}{\sqrt{a_1}\operatorname{erf}(\beta_1/\sqrt{a_1})} \exp\left[-\beta_1^2\left(\frac{1}{a_1} - \frac{1}{a_2}\right)\right] = \varkappa_1 \frac{\lambda_3 (T_0 - F(\beta_2))}{\sqrt{a_3}\operatorname{erfc}(\beta_2/\sqrt{a_3})} \exp\left[-\beta_2^2\left(\frac{1}{a_3} - \frac{1}{a_2}\right)\right], \tag{16}$$

$$\operatorname{erf}(\beta_1/\sqrt{a_2}) = \operatorname{erf}(\beta_2/\sqrt{a_2}) - \frac{\lambda_2 \sqrt{a_3}}{\lambda_3 \sqrt{a_2}} \frac{F(\beta_2) - T_c}{T_0 - F(\beta_2)} \operatorname{erfc}(\beta_2/\sqrt{a_3}), \tag{17}$$

$$F(\beta_2) = T_* = T_c - \frac{T_0 - T_c}{\psi - 1}, \tag{18}$$

where

$$\psi = \gamma \frac{\operatorname{erfc}(\beta_2/\sqrt{a_3}) \exp(\beta_2^2/a_3)}{\operatorname{erfc}(\beta_2/\sqrt{k_3}) \exp(\beta_2^2/k_3)}, \quad \gamma = \frac{\lambda_2 \sqrt{k_3 a_3}}{\lambda_3 k_2}. \tag{19}$$

Using the last inequality (1), we can easily show that $\psi(\beta_2)$ is a monotonically increasing function satisfying the condition:

$$1 < \gamma < \psi(\beta_2) < \gamma \frac{\sqrt{a_3}}{\sqrt{k_3}}.$$

It follows from these inequalities and from (18) that $F(\beta_2) < T_0$. Figure 3 shows the correlation between β_1 and β_2 plotted according to Eqs. (16), (17). The shape of these curves corresponds to the above relations because, in accordance with (1):

a) the function

$$F_1 = \frac{\exp\left[-\beta_1^2\left(\frac{1}{a_1} - \frac{1}{a_2}\right)\right]}{\operatorname{erfc}(\beta_1/\sqrt{a_1})}$$

has a unique local minimum and tends to infinity for $\beta_1 \to 0$, $\beta_1 \to \infty$;

b) the functions

$$F_2 = \frac{T_0 - F(\beta_2)}{\operatorname{erfc}(\beta_2/\sqrt{a_3})} \exp\left[-\beta_2^2\left(\frac{1}{a_3} - \frac{1}{a_2}\right)\right];$$

$$F_3 = \operatorname{erf}\left(\frac{\beta_2}{\sqrt{a_2}}\right) - \frac{\lambda_2 \sqrt{a_3}}{\lambda_3 \sqrt{a_2}} \frac{F(\beta_2) - T_c}{T_0 - F(\beta_2)} \exp\left[-\beta_2^2\left(\frac{1}{a_2} - \frac{1}{a_3}\right)\right] \operatorname{erfc}(\beta_2/\sqrt{a_3})$$

are monotonically increasing with respect to β_2.

If an intermediate freezing zone is to appear, the inequality $T_* > T_k$ has to be fulfilled, or we have to obtain, in accordance with (17), the relation $\beta_1 < \beta_2$. Let us consider the three cases:

a)
$$\frac{T_0 - T_c}{T_{on} - T_c} < \frac{\lambda_2}{\lambda_3} \frac{\sqrt{k_3 a_3}}{k_2}, \tag{20}$$

b)
$$\frac{\lambda_2}{\lambda_3} \frac{\sqrt{k_3 a_3}}{k_2} < \frac{T_0 - T_c}{T_{on} - T_c} < \frac{\lambda_2 a_3}{\lambda_3 k_2}, \tag{21}$$

$$\text{c)} \qquad \frac{\lambda_2 a_3}{\lambda_3 k_2} < \frac{T_0 - T_c}{T_{on} - T_c}. \tag{22}$$

The simplest are the variants a) and c). In the first case the intersection of curve 1 (Fig. 3) with curves type 4, 5 occurs at a unique point of the region $\beta_2 > \beta_1$ or $T_* > T_k$. This is the case of three zones. When (22) is fulfilled, the unique root of the system (16), (17) is unambiguously determined, and $\beta_1 > \beta_2$, $T_* < T_k$, i.e., three zones cannot form then. Instead of two movable boundaries, one forms, separating the melted from the frozen zone. The conditions on this boundary $s = 2\beta\sqrt{t}$ are written in the following way:

$$T_1 = T_3 = T_c \quad W_1 = W_3 = W_c,$$

$$\frac{\lambda_1}{\sqrt{a_1}} \frac{T_c - T_x}{\operatorname{erf}(\beta/\sqrt{a_1})} \exp(-\beta^2/a_1) = \frac{\lambda_3}{\sqrt{a_3}} \frac{T_0 - T_c}{\operatorname{erfc}(\beta/\sqrt{a_3})} \exp(-\beta^2/a_3) + \sqrt{k_3}\varkappa\rho\Delta W \frac{\exp(-\beta^2/k_3)}{\operatorname{erfc}(\beta/\sqrt{k_3})}. \tag{23}$$

The left-hand side is a function decreasing with respect to β, the right-hand side is increasing; Eq. (23) therefore has a unique root. The greatest complication is encountered in the case of b). Here two variants are possible: $\beta_1^* \gtrless \beta_2^*$, where β_1^* is the coordinate on the β_2 axis of the point of intersection of curve (16) with the bisector $\beta_1 = \beta_2$, β_2^* is the coordinate on the β_2-axis of the intersection of curve (17) with $\beta_1 = \beta_2$ (Fig. 3). When the condition $\beta_1^* > \beta_2^*$ is fulfilled, three zones form, when $\beta_1^* < \beta_2^*$, there is no freezing layer. In the last variant the coefficient β is again found from relation (23).

4. Analysis of the Obtained Solution. In accordance with (3), (12) the summary moisture in the intermediate layer is calculated in the following way:

$$W_s(x, t) = \int_{t_*}^{t} \partial W_s/\partial t dt + W_{on}(T_*) = \int_{t_*}^{t} k_2 \frac{\partial^2 W_2}{\partial x^2} dt + W_{on}(T_*) =$$

$$= k_2 \frac{\Delta W}{\Delta T} \int_{t_*}^{t} \frac{\partial^2 T_2}{\partial x^2} dt + W_{on}(T_*) = \frac{k_2}{a_2} \frac{\Delta W}{\Delta T} \int_{t_*}^{t} \frac{\partial T_2}{\partial t} dt + W_{on}(T_*) = \frac{k_2}{a_2} \frac{\Delta W}{\Delta T} (T_2 - T_*) + W_{on}(T_*).$$

Here, t_* is the instant at which at the given point x the soil temperature attains the value T_*, after which the phase transition from water to ice begins. It should be noted that in the last expression the sign of the first term is negative, i.e., in the freezing zone (the same as in the melted one) the summary moisture decreases in the direction of the frozen zone (see Fig. 2). The jump of W_s occurs at the boundary s_1. The value of the summary moisture in the frozen zone W_s is a constant and is determined in the following way:

$$W_s^1 = \frac{k_2}{a_2} \frac{\Delta W}{\Delta T} (T_c - T_*) + W_{on}(T_*) - [W_s]_1,$$

where $W_{on}(T_*)$ is determined from (3), and

$$[W_s]_1 = -\frac{k_2}{\sqrt{\pi a_2}\beta_1} \frac{\Delta W}{\Delta T} (T_* - T_c) \frac{\exp(-\beta_1^2/a_2)}{[\operatorname{erf}(\beta_2/\sqrt{a_2}) - \operatorname{erf}(\beta_1/\sqrt{a_2})]};$$

$$W_s^1 = \frac{\Delta W}{\Delta T} (T_* - T_c) \left\{ 1 - \frac{k_2}{a_2} + \frac{k_2}{\sqrt{\pi a_2}\beta_1} \frac{\exp(-\beta_1^2/a_2)}{\left[\operatorname{erf}\left(\frac{\beta_2}{\sqrt{a_2}}\right) - \operatorname{erf}\left(\frac{\beta_1}{\sqrt{a_2}}\right)\right]} \right\} + W_c.$$

The summary moisture on the boundary s_1 after fracture (W_s^2) and on s_2 (W_s^3) (Fig. 2) is determined:

$$W_s^2 = \frac{\Delta W}{\Delta T} (T_* - T_c) \left(1 - \frac{k_2}{a_2} \right) + W_c, \quad W_s^3 = W_{on}(T_*) = \frac{\Delta W}{\Delta T} (T_* - T_c) + W_c.$$

When only two zones form, the function W_s has a jump at the point $s = 2\beta\sqrt{t}$, and then

$$W_s^2 = W_s^3 = W_c, \quad W_s^1 = \frac{\sqrt{k_3}\Delta W}{\sqrt{\pi}\beta} \frac{\exp(-\beta^2/k_3)}{\operatorname{erfc}(\beta/\sqrt{k_3})} + W_c,$$

where β is the root of Eq. (23).

5 Comparison of the Obtained Solution with the Experimental Data. As shown above, the effect of formation of an intermediate layer was noted experimentally only in soils that con-

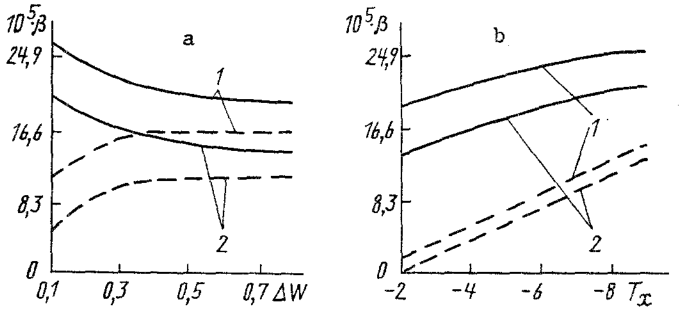

Fig. 4. Dependence of the coefficients β_1, β_2 on ΔW, kg/kg (a); T_x, °C (b): a) $k_2 = 5.5\cdot10^{-9}$ m²/sec, $k_3 = 2.8\cdot10^{-8}$ m²/sec; 1) $T_0 = 3$°C, $T_x = -10$°C; 2) $T_0 = 5$°C, $T_x = -6$°C; b) $k_2 = 5.5\cdot10^{-9}$ m²/sec, $k_3 = 1.7\cdot10^{-8}$ m²/sec, $\Delta T = 1.7$°C; 1) $T_0 =$ 1°C, 2) $T_0 = 3$°C.

tained a sufficient amount of clayey particles. Their increase leads to a decrease of k_2 and an increase of ΔT [1, 2]. It can be seen from (20) that with other conditions being equal, such a change of k_2 and ΔT leads to a more probable formation of three zones. Conversely, upon transition to coarsely disperse soils, e.g., sands, k_2 increases, $\Delta T \to 0$, i.e., according to (20) the freezing zone is bound to vanish. In a number of publications (e.g., [2]) it was noted that this zone becomes narrower when the initial moisture content of the specimens increases. Figure 4 presents the dependence of the coefficients β_1, β_2 on ΔW obtained in accordance with the suggested solution (β_1, β_2 are denoted by a dashed and a solid line, respectively). The difference $\Delta\beta = \beta_2 - \beta_1$, characterizing the thickness of the freezing layer, decreases with increasing ΔW, the same as in the experiments. In the same book it was shown that when the cooling temperature T_x is higher (softening of the freezing regime), the freezing zone becomes wider. It can also be seen from the theoretical dependences (Fig. 4b) that with higher T_x the value of $\Delta\beta$ increases. A comparison of the theoretical dependences of moisture content at the boundary of freezing $W_i(s_2)$ on some parameters with the experimental data [9] showed that here there is complete coincidence at the qualitative level: a) increase of W_i when W_0 increases; b) increase of W_i when T_x increases; c) W_i is constant in the process of each actual experiment. In all the calculations the following values of the initial parameters were adopted: $\varkappa = 335$ kJ/kg, $\rho = 1500$ kg/m³, $\lambda_1 = \lambda_2 = \lambda_3 =$ 1.03 W/(m·deg), $c_1 = c_2 = c_3 = 1745$ kJ/(m³·deg).

On the whole, the analysis of the presented results enables us to assert that the suggested self-simulating solution describes fairly accurately the change of phase composition and the migration processes accompanying the freezing of disperse soils.

NOTATION

a, thermal diffusivity of the soil, m²/sec; $[A]_i$, difference between the values of A ahead of the i-th boundary and behind it; v_i, speed of migration of the i-th boundary, i = 1, 2; c, heat capacity of the soil, J/(m³·deg); k, diffusion coefficient of the moisture, m²/sec; L, iciness of the soil, kg/kg; s_2, s_1, coordinates of the boundary of the beginning and end of the phase transition of the moisture, m; t, time, sec; T, soil temperature, °C; T_*, temperature at the mobile boundary separating the freezing from the melted zone, °C; T_n, temperature of the beginning of the phase transition of the moisture in the soil, °C; W, moisture content, kg/kg; W_k, moisture content of the frozen zone, kg/kg; W_0, initial moisture content, kg/kg; W_s, summary moisture, kg/kg; W_i, moisture content at the boundary of freezing, kg/kg; $W_n(T)$, dependence of the equilibrium content of nonfrozen water on the temperature, kg/kg; x, spatial coordinate, m; β_1, β_2, coefficients characterizing the speed of migration of the boundaries s_1, s_2, respectively, m/sec$^{1/2}$; $\Delta T = T_n - T_k$; $\Delta W = W_0 - W_k$; κ, latent heat of the phase transition water–ice, J/kg; λ, thermal conductivity of the soil, W/(m·deg); ρ, volumetric mass of the soil skeleton, kg/m³; λ_i, c_i, a_i, k_i, thermal conductivity, W/(m·deg), heat capacity, J/(m³·deg), thermal diffusivity, m²/sec, diffusion coefficient, m²/sec, respectively, of the moisture in the i-th zone, i = 1, 2, 3; T_k, temperature of the conclusion of the phase transition of the moisture in the soil, °C.

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THE COOLING OF A SALT SOLUTION

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Self-similar formulation is used to demonstrate the possibility of a regime involving formation of a zone with two-phase state. The boundary is found between crystallization regimes with abrupt phase transition front and with extended mixture zone.

We will consider a generalization of the classical Stefan problem (see, for example, [1, 2]) of one-dimensional passage of a planar crystallization front through a cooled liquid. We will assume that in the liquid (water) there is dissolved a small quantity of material (salt) which does not enter the solid phase upon crystallization. Since the dissolved material decreases the phase transition temperature and is retained in the liquid phase upon crystallization, it is necessary to solve a mixed thermodiffusion problem, similar to that of crystallization of a binary alloy [2, 3]. As is well known, analysis of the self-similar solution [3] has shown that it becomes physically absurd at some parameter range, since in the melt zone ahead of the phase transition front the temperature of the melt proves to be lower than the local crystallization temperature. This occurs because for a sufficiently small diffusion coefficient the concentration ahead of the front decreases very rapidly and the corresponding phase transition temperature increases with removal from the front more rapidly than the local temperature. This effect has been termed "diffusion" supercooling [3]. To construct a solution free of this shortcoming the concept and model of a two-phase zone was introduced, which on the average describes crystallization with formation of dendrites in the case of supercooling [4, 5]. This model, well known in metallurgy, has apparently not been applied to freezing processes in salt solutions, in particular, to freezing of soil moisture. Meanwhile, formation of a two-phase zone here can lead to significant quantitative and even qualitative effects. The goal of the present study is the formulation of a corresponding mathematical model and determination of the boundaries of problem parameters separating qualitatively different freezing regimes.

The fact that the classical "Stefan" regime may not be realizable is illustrated by Fig. 1, which gives an example of calculation of the self-similar solution of the problem of freezing of an aqueous solution of NaCl assuming the presence of a phase transition front. It is clear that supercooling of the solution ahead of the freezing front occurs. We will now assume that between the impurity-free ice and the liquid solution there exists an intermediate zone with a two-phase state, in which ice and the solution coexist in a state of local thermodynamic equilibrium, so that their temperatures are equal to each other and the phase transition temperature for the local value of the salt concentration in the solution. Such behavior has been observed in experiment [6], although it has been calculated only with neglect of salt diffusion.

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